.Chaotic System for Self Synchronizing Doppler Measurement

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In a radar system, it is necessary to measure both range and velocity of a target. The movement of the target causes a Doppler shift of the radar signal, and the size of the Doppler shift is used to measure the velocity of the target. In this work, I simulate a chaotic drive-response system which detects a Doppler shift in a chaotic signal. The response system can detect Doppler shifts in more than one signal at a time.

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Both radar and sonar systems transmit a signal which is reflected from an object, and the reflected signal is detected by a receiver. The round trip time (transmit-reflect-receive) is used to measure the distance to the object. The resolution with which distance can be measured is increased by increasing the bandwidth of the signal, which suggests the use of broad band chaotic signals.

If the object is moving toward or away from the receiver, the received signal is also Doppler shifted. The Doppler shift is proportional the velocity of the object.

Some types of chaotic systems may be synchronized to each other by sending a signal from a drive system to a response system. The response system is arranged so that it synchronizes to the drive system. While in some cases synchronization means that the response system does the same thing as the drive system, in the work described here only the phase of the response system synchronizes to the drive

system. When the driving signal is Doppler shifted, the response system phase synchronizes to this Doppler shifted signal. A frequency reference signal is also maintained in the response system, so the size of the Doppler shift may be measured by comparing the response system frequency to the frequency reference.

INTRODUCTION

In the fields of radar and sonar, transmitted signals that are reflected from objects are used to determine the distance to and radial velocity of those objects [1]. Usually, periodic signals or combinations of periodic signals have been used for these applications, but other types of signals such as noise or chaos have also been explored. Chaotic signals may have a large bandwidth, which is useful in increasing the precision with which distances may be measured. To understand this principle, think of a pulse or a wave packet: the more the wave packet is localized in time, the greater the spread of frequencies used to create the wave packet The application of chaos to radar or sonar has been considered by several groups [2-6]. To measure distance in radar, one actually measures a delay time, so signals with broader bandwidths increase the precision with which distance may be measured. The work so far on applying chaos to radar has been concentrated on the properties of the chaotic waveform itself, either studying how the broad bandwidth of chaos gives range resolution [7], or studying how chaotic waveforms may be separated from other chaotic waveforms or interference [2-6]. In this paper, I concentrate on the structure of the transmitter and receiver, rather than on the waveform itself. I describe how the properties of a self synchronizing chaotic system may be used to extract a Doppler shift from a chaotic signal without having a copy of the transmitted signal for reference.

RADAR AND SONAR BASICS

Radar (or sonar) signals may also be used to measure the velocity of a target towards or away from the observer [1]. The range to the target as a function of time is $R(t) = R_0 + v_r t$, where R_0 is the initial range and v_r is the radial component of the velocity. Assuming the target velocity is much slower than the signal velocity (true for radar), the transmitted signal at time t_1 then travels a total path length of $2R(t_1)$, for a phase change of $4\pi R(t_1)/\lambda$, where λ is the wavelength. The Doppler frequency is the derivative with respect to time of this phase shift, which may be simplified to yield

$$f_d = \frac{2f}{c} v_r \tag{1}$$

where f is the signal frequency f_d is the resulting Doppler frequency, and c is the signal velocity. Eq. (1) shows that if the transmitted signal contains many frequencies, then there will also be many Doppler frequencies.

An ambiguity diagram is used to measure the ability of a radar signal to measure delay and Doppler shift [1]. An ambiguity diagram represents the output of a matched filter for the signal. A matched filter is a linear filter that given an input signal with additive Gaussian noise, produces the optimum signal to noise ratio for that signal at the output [8]. For a radar signal, taking the cross correlation between the transmitted and received signals is mathematically equivalent to applying a matched filter to the received signal. The function that is plotted for the ambiguity function is

$$\left|\chi(T_R, f_d)\right|^2 = \left|\int_{-\infty}^{\infty} u(t) u^*(t + T_R) e^{2\pi j f_d t} dt\right|^2 \tag{2}$$

where $u(t)$ is the signal under consideration and T_R is a time delay.

For a narrow band signal, a single Doppler frequency f_d may be used in eq. (2), but for a broad band signal, a band of frequencies defined by eq. (1) is necessary [7, 9]. To calculate the ambiguity function, the signal $u(t_i)$ is first Fourier transformed to produce $U(f_j)$. The subscripts i and j are used to indicate that a discreetly sampled signal is used. The frequency components of $U(f_i)$ are shifted according to $\tilde{f}_i = f_i(1+\delta)$, where $\delta = v_r/c$. The product $U(f_i)U^*(\tilde{f}_i)$ is taken and inverse Fourier transformed to produce $|\chi(T_R, \delta)|^2$, the wideband ambiguity function. In a following section I will plot an ambiguity function for a particular chaotic signal is produced by a chaotic system I have designed.

CHAOS FOR RADAR

There are several ways in which a self-synchonizing chaotic system may be useful for radar. First, chaotic signals are broad band, so they have good range resolution. A self synchronizing chaotic system can extract information without a stored reference signal, so it might be useful for a bistatic radar system, on which transmitter and receiver are at different locations. There are other reasons to use chaotic signals for radar, but they are not considered in this paper.

The broad band nature of the chaotic signal does cause some difficulties for estimating Doppler shifts. If the chaotic signal is digitized upon reception, then one can proceed as in the section above on measuring ambiguity diagrams, but it may be that a completely analog receiver is necessary. In the common analog method to measure Doppler shifts, the received signal is multiplied by a reference signal which has no Doppler shift. The Doppler frequency f_d appears as a difference frequency. If the same procedure is attempted for the broad band chaotic signal, a broad band of difference frequencies is produced, so it is not possible to read a single Doppler frequency. To correct this problem, a chaotic response system was designed that converts the broad band chaotic signal into a narrow band signal at a lower frequency.

CHAOTIC SYSTEM

DRIVE SYSTEM

The chaotic system used in this paper was based on a piecewise linear Rossler system [10, 11]. This particular system has motion on 4 different time scales, so I call it the 4-frequency piecewise linear Rossler system. The 4-frequency Rossler system is based on the 2-frequency Rossler system.

The 2-frequency Rossler system was originally designed as a noise robust communication system [10, 12]. The reason that the combination of a fast Rossler oscillator and a slow damped periodic system is noise robust was studied in [13]. The slow periodic system modulates the frequency of the fast Rossler system, so that the combined system has a larger bandwidth than the Rossler system alone.

If one observes the output of the 2-frequency Rossler system for a short time, it still looks like a narrow band system, since the frequency varies only slowly. It seemed that a system which looked broad band even on short time scales might be more desirable for radar or sonar, so the 4-frequency Rossler was designed to have a large instantaneous bandwidth. The ambiguity diagram for the 4-frequency Rossler system shown below in fig. 3 does have smaller range sidelobes than the ambiguity diagram for the 2-frequency Rossler. It was shown in [13] that the noise robustness of the 2-frequency Rossler depended on having a large ratio of the fastest to the slowest frequencies, so the 4-frequency Rossler was designed with a large range of frequencies.

Figure 1 is a block diagram of the 4-frequency Rossler system. This system was designed to have an output signal which was broad band, but whose bandwidth could easily be controlled. The drive system was composed of a chaotic piecewise linear Rossler system which drove 3 passive damped periodic systems. The 3 periodic systems provided signals which varied the time scale of the chaotic Rossler system. The time constants of the 3 periodic systems were chosen to be roughly incommensurate with each other. The entire drive system was described by:

$$
\begin{aligned}
\frac{dx_1}{dt} &= -\gamma_d \lambda(x) \alpha_1 \left(.05x_1 + 0.5x_2 + x_3 + 0.1x_4\right) \\
\frac{dx_2}{dt} &= -\gamma_d \lambda(x) \alpha_1 \left(-x_1 - 0.13x_2\right) \\
\frac{dx_3}{dt} &= -\gamma_d \lambda(x) \alpha_1 \left(-g(x_1) + x_3\right) \\
\\
\frac{dx_4}{dt} &= -\gamma_d \alpha_2 \left(0.02x_4 + 0.5x_5 + 0.5|x_1|\right) \\
\frac{dx_5}{dt} &= -\gamma_d \alpha_2 \left(-x_4 + 0.02x_5\right) \\
\\
\frac{dx_6}{dt} &= -\gamma_d \alpha_3 \left(0.02x_6 + 0.5x_7 + 0.5|x_1|\right) \\
\frac{dx_7}{dt} &= -\gamma_d \alpha_3 \left(-x_6 + 0.02x_7\right) \\
\\
\frac{dx_8}{dt} &= -\gamma_d \alpha_4 \left(0.02x_8 + 0.5x_9 + 0.5|x_1| + 0.01s_x\right) \\
\frac{dx_9}{dt} &= -\gamma_d \alpha_4 \left(-x_8 + 0.02x_9\right) \\
s_x &= \sin\left(\gamma_d \omega_4 t\right) \\
\\
\lambda(x) &= 1 + 0.2\left(x_4 + x_6 + x_8 + 0.1\right) \\
g(x) &= \begin{Bmatrix} 0 & x < 3 \\ 15(x-3) & x \geq 3 \end{Bmatrix}
\end{aligned} \tag{3}
$$

where $a_1 = 1.0$, $a_2 = 0.565$, $a_3 = 0.1$, and $a_4 = 0.051$. The α constants define the time scales for the different parts of the system. The signal s_x provides a phase and frequency reference for the slowest part of the drive system, where $\omega_4 = 0.00575$ Hz is the center frequency of the x_8-x_9 part of the drive system for $\gamma_d = 1.0$. The constant γ_d is used to slightly vary the overall time constant to simulate a Doppler shift. For no Doppler shift, $\gamma_d = 1.0$. The function $\lambda(x)$ varies the time scale of the chaotic system of x_1-x_3 .

Eqs. (3) is integrated with a 4^{th} order Runge-Kutta integration routine with a time step of 0.4 s. The signal x_2 is extracted from eqs. (3) to use as a driving signal for the response system. Figure 2 is a power spectrum of the x_2 signal.

AMBIGUITY DIAGRAM FOR CHAOS

The maximum achievable time and frequency resolution for the signal x_2 using a matched filter can be seen by plotting the ambiguity diagram (fig. 3) for x_2 , as defined by eq. (2). The ambiguity surface is seen to have large sidelobes along the T_R axis, so it is not the ideal waveform for detecting the range to a target. The large sidelobes can obscure closely spaced targets [1]. The ambiguity surface does not have any sidelobes along the frequency shift axis, so it is useful for detecting Doppler shifts. To produce this ambiguity diagram, a 40,000 point time series of x_2 was used for the signal *u(t)*. Figure 3 is a useful way to characterize the potential of the chaotic signal for measuring delays and Doppler shifts..

RESPONSE SYSTEM

The response system was not identical to the drive system. The purpose of the response system was to convert the broad band signal from the drive system into a narrow band signal (at lower frequency) from which the Doppler shift could be measured. Figure 4 is a block diagram of the response system. The response system was defined by

$$
\begin{aligned}
&\xi_d = x_2 + \eta \\
&\frac{dy_1}{dt} = -\lambda_r(y)\alpha_1\left(0.02y_1 + 0.5y_2\right) \\
&\frac{dy_2}{dt} = -\lambda_r(y)\alpha_1\left(-y_1 - \xi_d\right) \\
\\
&\frac{dy_3}{dt} = -\alpha_2\left(0.1y_3 + 0.5y_4 + 0.1\left|y_1\right|\right) \\
&\frac{dy_4}{dt} = -\alpha_2\left(-y_3 + 0.1y_4\right) \\
\\
&\frac{dy_5}{dt} = -\alpha_3\left(0.1y_5 + 0.5y_6 + 0.1\left|y_3\right|\right) \\
&\frac{dy_6}{dt} = -\alpha_3\left(-y_5 + 0.1y_6\right) \\
\\
&\frac{dy_7}{dt} = -\alpha_4\left(0.1y_7 + 0.5y_8 + 0.1\left|y_5\right|\right) \\
&\frac{dy_8}{dt} = -\alpha_4\left(-y_7 + 0.1y_8\right) \\
\\
&\lambda_r(y) = 1 + 0.05\left(y_3 + y_5 + 0.5\right)
\end{aligned}
\qquad (4).
$$

The α*'s* are the same as in eq. (3). The actual driving signal ξ_d is a sum of x_2 from the driving system and an interfering signal η , which may be noise or another chaotic signal. The function $\lambda_r(y)$ varies the center frequency of the y_1-y_2 system.

The response system of eqs. (4) is not identical to the drive system of eqs. (3), so it can't exhibit exact synchronization; rather, it phase synchronizes to the drive system [14]. It has been shown before that response systems with structures similar to eqs. (4) can maintain good synchronization even when large amounts of noise (larger than the drive signal) are added to the drive signal [10-13, 15].

As a Doppler detector, the response system is used to convert the broad band x_2 signal to the much narrower band y_7 signal. Since y_7 is narrow, there is only one Doppler frequency to find. The y_7 signal is filtered to remove any DC offset, multiplied by a single frequency reference signal s_y, and filtered to produce a Doppler signal z_2

$$\begin{aligned} s_y &= \sin(\omega_4 t) \\ \frac{dz_1}{dt} &= -\frac{\alpha_4}{100.0}\left(y_7 s_y + z_1\right) \\ \frac{dz_2}{dt} &= \frac{dz_1}{dt} - 10^{-5} z_2 \end{aligned} \tag{5}$$

Figure 5 shows the power spectrum of y_7 when the Doppler shift factor γ_d in eqs. (3) is –0.99, 1.0, or 1.01. It can be seen that Doppler shifting the driving signal shifts the frequency of y_7 up or down. Figure 6 shows the Doppler signal z_2 when the Doppler factor γ_d = 1.01. The frequency of the Doppler signal is 5.75×10^{-5} Hz, which is 1% of the frequency of y_7 , as is expected. A Doppler shift of 1% is actually much larger than anything that would be seen in a radar signal (see eq. (1)), but the shift was made large in order to shorten the simulation time. A 1% Doppler shift is less extreme for sonar.

INTERFERENCE EFFECTS

It is rare that only a single target is present to scatter a radar signal. The most common types of interference in radar are reflections from other targets, which may be moving or stationary [1]. Fixed objects, such as the ground or buildings, usually provide reflections that are larger than the signal reflected from the target. The radar system must be able to separate the target reflection from these different signals.

Figure 7 shows the Doppler signal z_2 when a chaotic signal from eqs. (3) with no Doppler shift (and different initial conditions) was added to x_2 (which had a Doppler shift of 1.01). The added signal had an amplitude 5 times the amplitude of x_2 , or 25 times the power. The Doppler peak corresponding to a 1% Doppler shift is still clearly visible in Fig. 7. The Doppler signal z_2 is produced in a way that produces no signal if no Doppler shift is present, so there is no peak for the signal with no Doppler shift. It is frequently the case in radar that one wants to detect only moving targets.

There may also be other moving targets reflecting the radar signal, and it would be useful to be able to distinguish them Figure 8 shows the power spectrum of the Doppler signal z_2 when an interfering signal with a Doppler shift of 1.005 (and different initial conditions) was added to x_2. The amplitude of the interfering signal was 4 times the amplitude of x_2 , or 16 times the power. Peaks corresponding to both signals can be seen in Fig. 8, showing that both Doppler shifts were detected.

CONCLUSIONS

A self synchronizing chaotic system that could detect a Doppler shift in a chaotic system was described. Similar types of systems have been built as analog circuits [10, 12, 15], so building this chaotic system would not be difficult. Implementation of this idea at microwave frequencies will have to wait for suitable high frequency chaotic circuits, but such systems are being developed [16, 17]. It was also shown that the self-synchronized chaotic response system also worked when simple but common types of interference were added to the chaotic signal. Some theory has been studied on the origin of this resistance to noise [13].

While this chaotic system detected a Doppler shift, it doesn't detect the delay between transmission and reception of a signal, which is necessary to determine the range to a target. It was shown previously how to use control [15] or modulation [12] techniques to impose information on a chaotic signal for a very similar type of chaotic system, so such techniques could possible be used to add time coding signals to the chaotic signal, allowing the determination of delay time. It is also possible that this system could be used purely as a simple motion detector, since it produces an output signal only when the received signal has been Doppler shifted.

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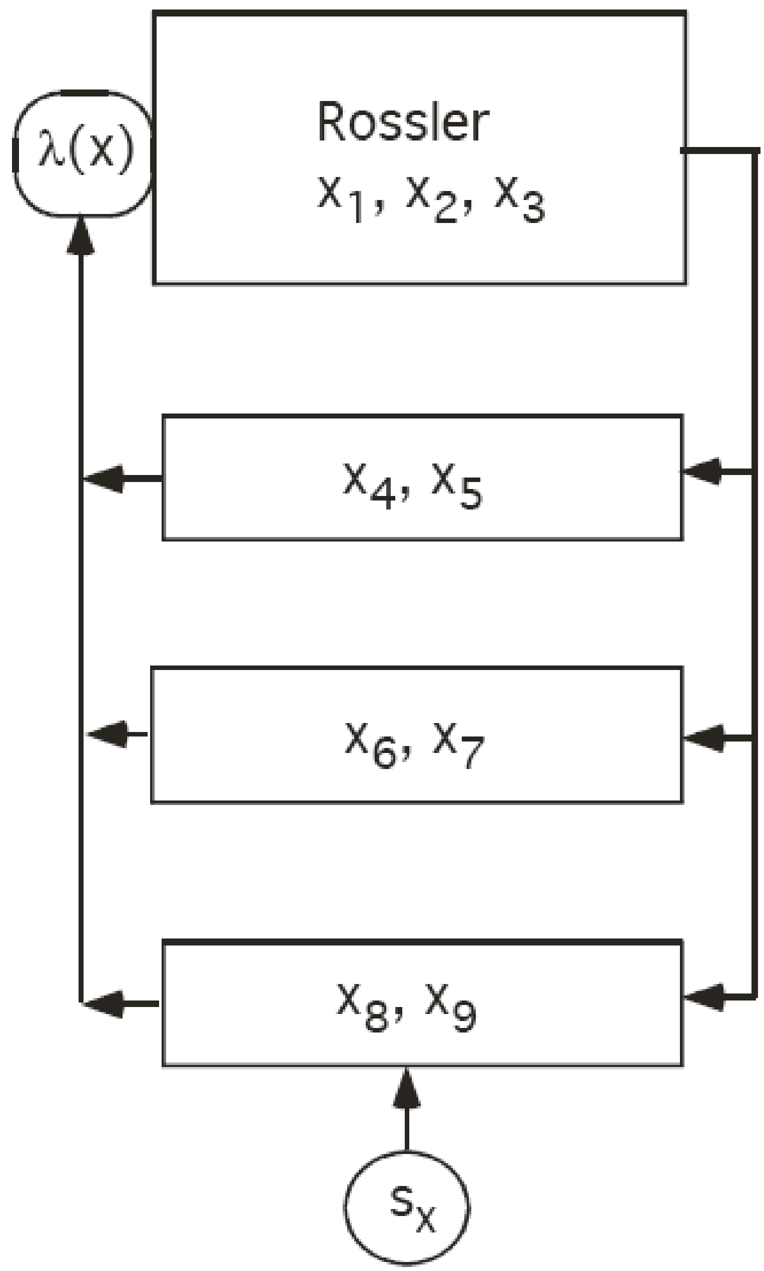

Fig. 1. Block diagram of drive system.

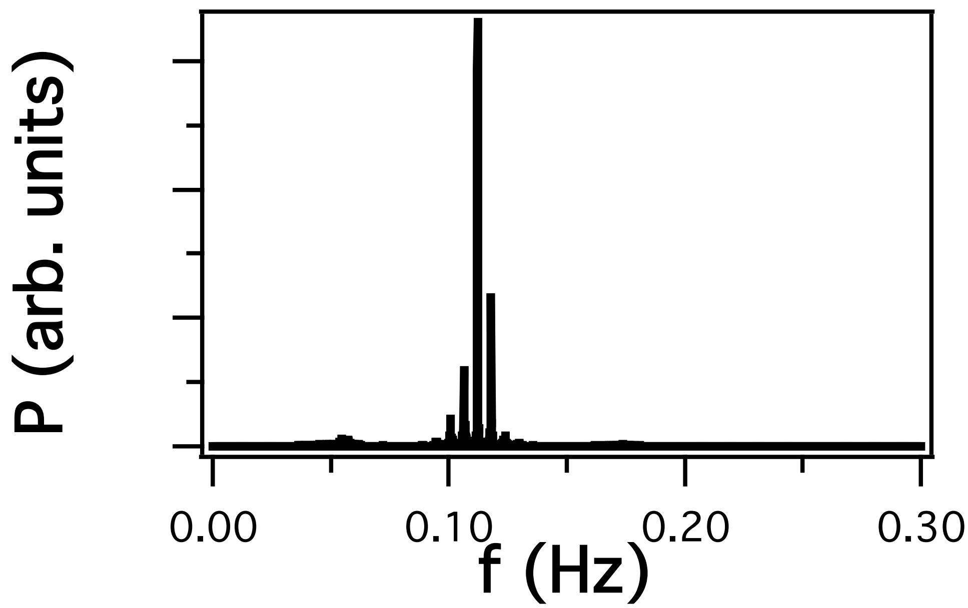

Fig. 2. Power spectrum of the x_2 signal from eqs. (3) The spectrum was generated from a time series of 40,000 points sampled at 0.4 s/pt.

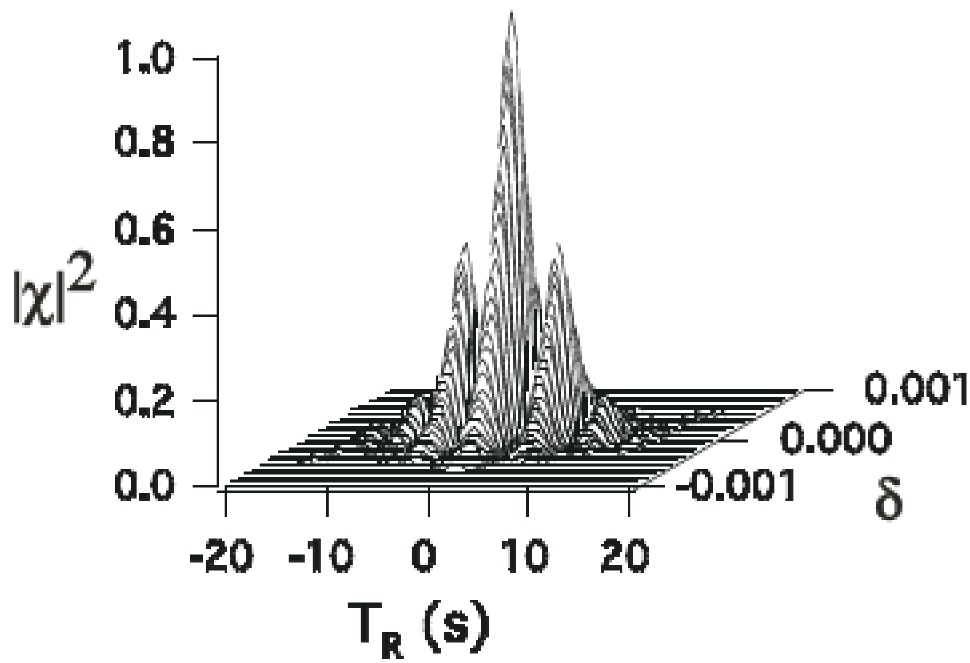

Fig. 3. Ambiguity function $|\chi|^2$ as defined in eq. (2) for the signal x_2. T_R is the delay time, while δ is the fraction by which the signal frequency is shifted. The diagram was generated from time series of 4000 points sampled at 0.4 s/pt, but the time axis of this plot covers only plus or minus 50 points.

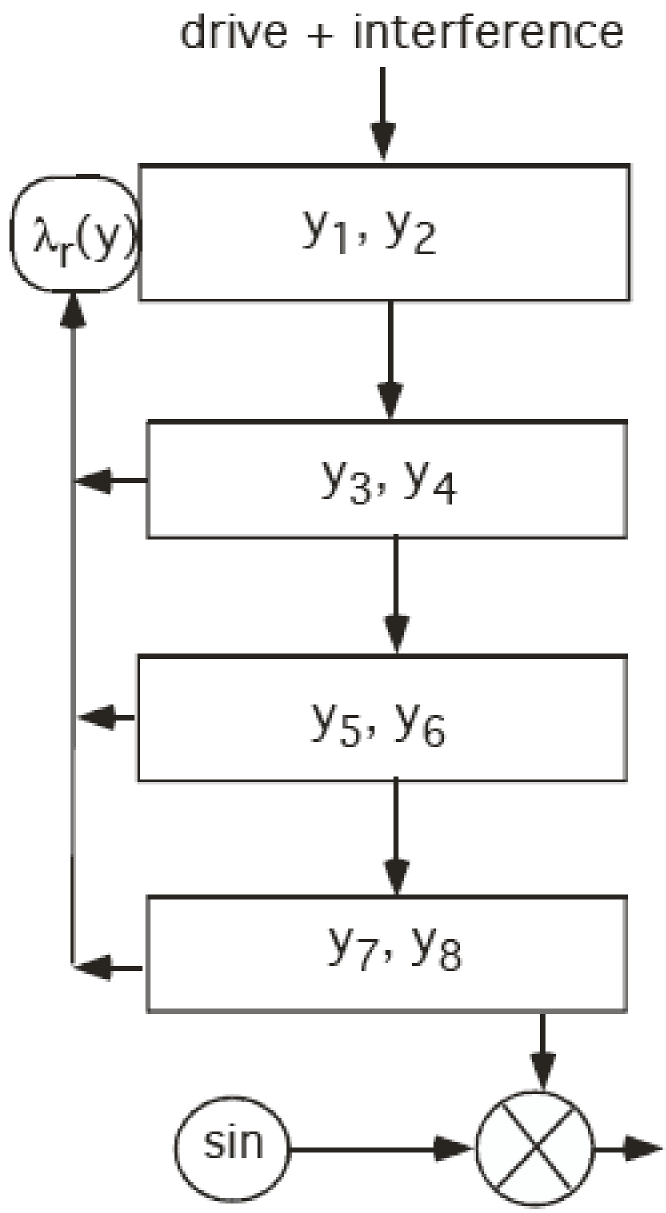

Fig. 4. Block diagram of the response system

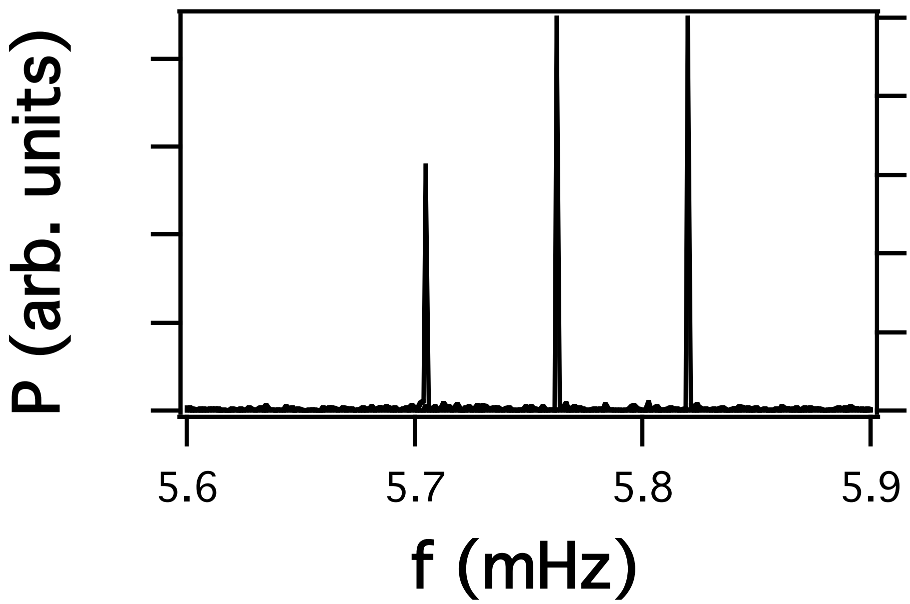

Fig. 5. Power spectrum of the output signal y_7 from eq. (4). The center line is the power spectrum when no Doppler shift is present in the transmitted signal x_2, the left peak is the power spectrum when the Doppler shift factor is 0.99 (or –1%) , and the right peak is the spectrum when the Doppler shift factor is 1.01 (or +1%). The time series for these spectra contained 40,000 points sampled every 50 integration time steps, for 20 s/pt.

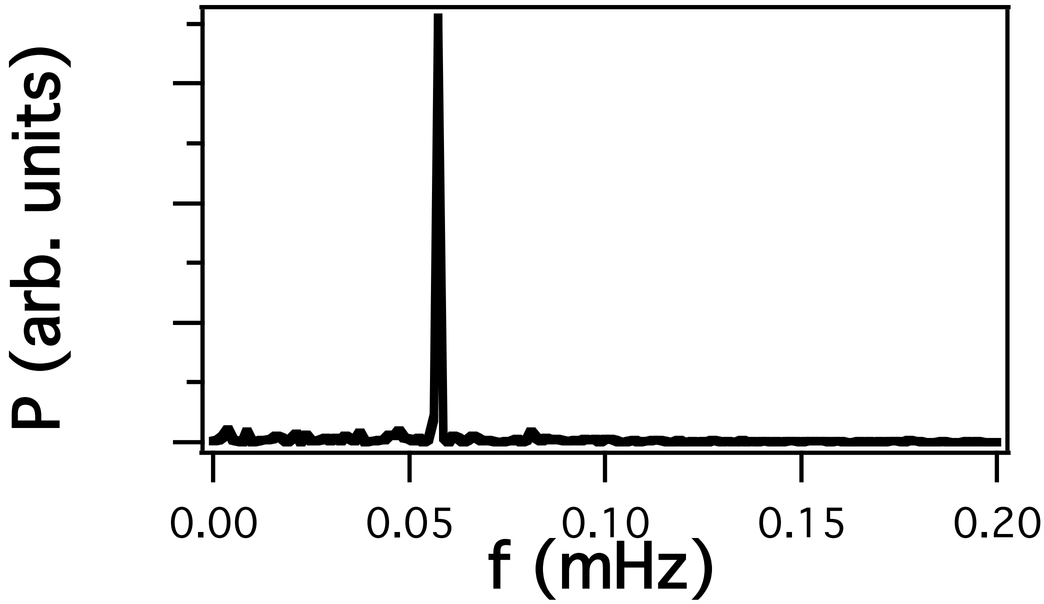

Fig. 6. Power spectrum of the Doppler signal z_2 from eq. (5) when the transmitted signal has a Doppler shift of +1%. The time series for this spectrum contained 40,000 points sampled every 50 integration time steps, for 20 s/pt.

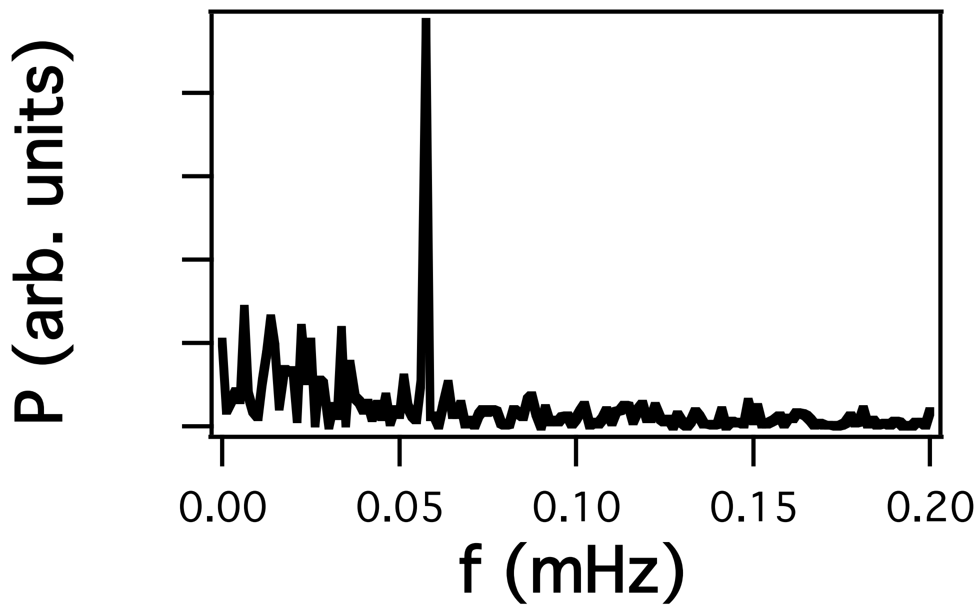

Fig. 7. Power spectrum of the Doppler signal z_2 from eq. (5) when an interfering chaotic signal with no Doppler shift and 5 times the amplitude of the x_2 signal is added to the x_2 signal, which has a Doppler shift of +1%. The time series for this spectrum contained 40,000 points sampled every 50 integration time steps, for 20 s/pt.

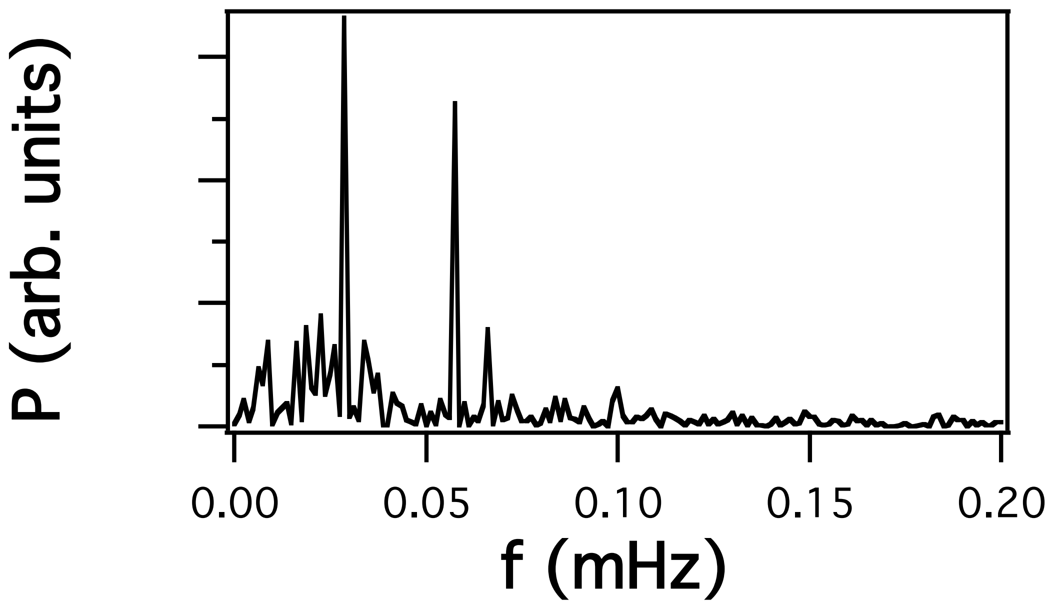

Fig. 8. Power spectrum of the Doppler signal z_2 from eq. (5) when an interfering chaotic signal with a Doppler shift of +0.5% and 4 times the amplitude of the x_2 signal is added to the x_2 signal, which has a Doppler shift of +1%. The time series for this spectrum contained 40,000 points sampled every 50 integration time steps, for 20 s/pt.